Atmospheric turbulence and the boundary layer

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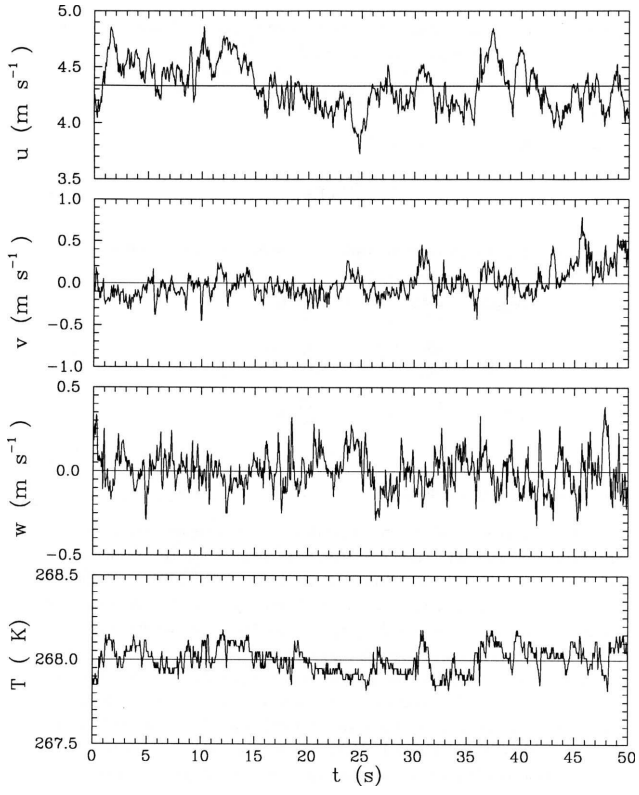
u_i and t as stochastic variables



(1) $$\zeta = \overline{\zeta} + \zeta'$$

Averaging, correlations, spectra

ensemble and time averaging

(2) $$\overline{\zeta}(\mathbf{x},t) = \frac{1}{N}\sum_{i=1}^{N}\zeta^{(i)}(\mathbf{x},t)\ ;\ \overline{\zeta}(\theta) = \frac{1}{\mathcal{T}}\int_{\theta-\mathcal{T}/2}^{\theta+\mathcal{T}/2}\zeta(t)\,\mathrm{d}t$$

correlation and spectrum (homogenous, steady conditions); x space or time, y wavenumber or frequency;

(3) $$R(x) = \overline{u'(x_0)u'(x_0+x)}$$

(4) $$E(y) = \frac{2}{\pi}\int_0^\infty R(x)\cos(yx)\,\mathrm{d}x\ ;\ R(x) = \int_0^\infty E(y)\cos(yx)\,\mathrm{d}y$$

Integral scales

(5) $$L = \frac{1}{\overline{u'^2}} \int_0^\infty R(r)\,\mathrm{d}r \; ; T = \frac{1}{\overline{u'^2}} \int_0^\infty R(t)\,\mathrm{d}t$$

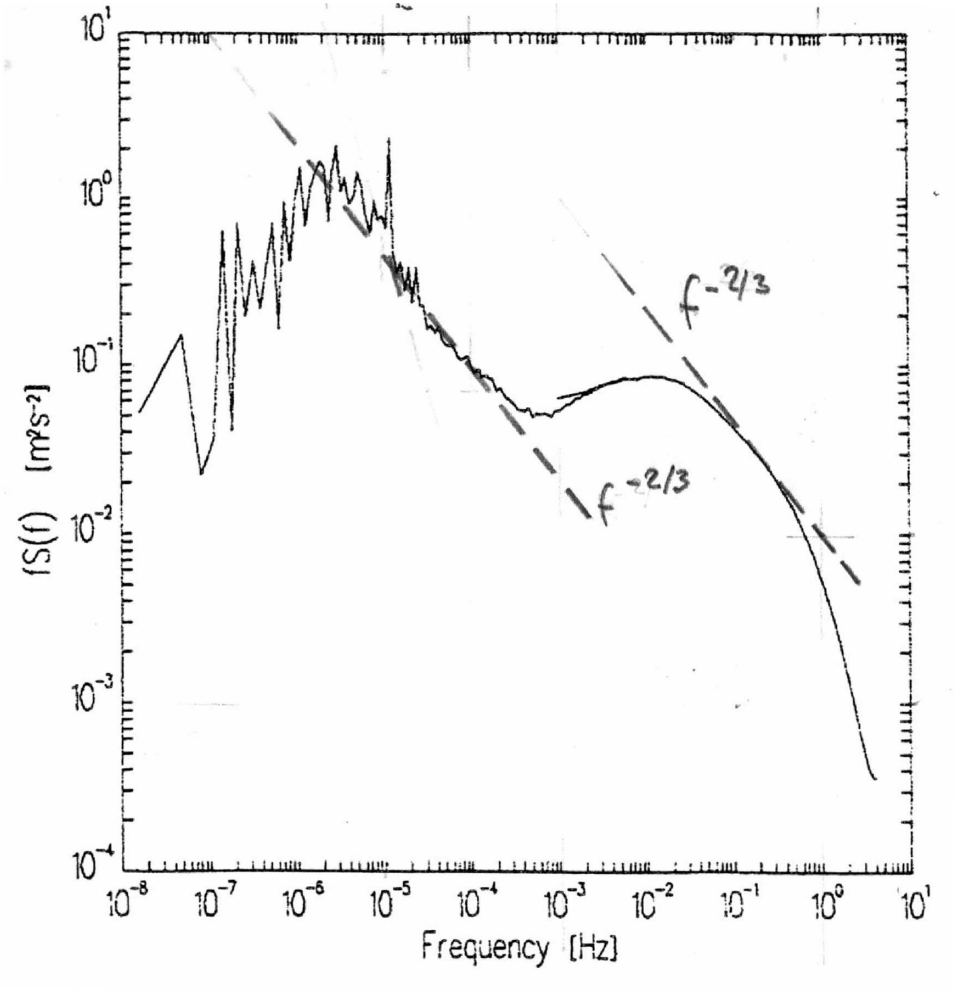
Note that, for instance,

(6) $$E(0) = \frac{2}{\pi} \int_0^\infty R(t)\,\mathrm{d}t = \frac{2}{\pi} T \overline{u'^2}$$

and

(7) $$R(0) \equiv \overline{u'^2} = \int_0^\infty E(y)\,\mathrm{d}y$$

Spectrum of the wind (near the ground)



NSE etc.

(8) $$\frac{\mathrm{D}u_i}{\mathrm{D}t} = -\frac{1}{\rho_f}\frac{\partial p_f}{\partial x_i} + \varepsilon_{ij3} f u_j + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \delta_{i3} g$$

(9) $$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j}$$

(10) $$\frac{\partial u_i}{\partial x_i} = 0$$

(11) $$\frac{\partial \zeta}{\partial t} = -\frac{\partial F_i}{\partial x_i}, \; F_i = u_i \zeta - \kappa_\zeta \frac{\partial \zeta}{\partial x_i}$$

(12) $$\frac{\mathrm{D}\zeta}{\mathrm{D}t} = \kappa_\zeta \frac{\partial^2 \zeta}{\partial x_j \partial x_j}$$

Hydrostatic pressure; geostrophic wind

(13) $$\frac{\partial p_a}{\partial x_3} = -\rho_a g$$

(14) $$\frac{\mathrm{D}u_i}{\mathrm{D}t} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} + \varepsilon_{ij3} f u_j + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \delta_{i3} g \frac{T}{T_a}$$

(15) $$\begin{aligned} - \quad \frac{1}{\rho_{00}}\frac{\partial p}{\partial x} + f v_g &= 0 \\ - \quad \frac{1}{\rho_{00}}\frac{\partial p}{\partial y} - f u_g &= 0 \end{aligned}$$

RANS etc

(16) $$\frac{\mathrm{D}\overline{u}_i}{\mathrm{D}t} = -\frac{1}{\rho_{00}}\frac{\partial \overline{p}}{\partial x_i} + \nu\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} + \delta_{i3}\frac{g}{\vartheta_{00}}\overline{\vartheta} - \frac{\partial \overline{u'_i u'_j}}{\partial x_j}$$

(17) $$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + \overline{u}_j \frac{\partial}{\partial x_j}$$

(18) $$\frac{\mathrm{D}\overline{\zeta}}{\mathrm{D}t} = \kappa_\zeta \frac{\partial^2 \overline{\zeta}}{\partial x_j \partial x_j} - \frac{\partial \overline{u'_j \zeta'}}{\partial x_j}$$

(19) $$\frac{\mathrm{D}u'_i}{\mathrm{D}t} + u'_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho_{00}}\frac{\partial p'}{\partial x_i} + \nu\frac{\partial^2 u'_i}{\partial x_j \partial x_j} + \delta_{i3}\frac{g}{\vartheta_{00}}\vartheta' - \frac{\partial(u'_i u'_j - \overline{u'_i u'_j})}{\partial x_j}$$

Velocity second order moments

$$
\begin{aligned}
\frac{\mathrm{D}\overline{u_i' u_k'}}{\mathrm{D}t} &= \left(-\overline{u_i' u_j'}\frac{\partial \overline{u}_k}{\partial x_j} - \overline{u_k' u_j'}\frac{\partial \overline{u}_i}{\partial x_j}\right) - \frac{\partial \overline{u_i' u_j' u_k'}}{\partial x_j} \\
&+ \frac{g}{\vartheta_{00}}(\delta_{k3}\overline{u_i'\vartheta'} + \delta_{i3}\overline{u_k'\vartheta'}) \\
&- \frac{1}{\rho_{00}}\left(\overline{u_k'\frac{\partial p'}{\partial x_i}} + \overline{u_i'\frac{\partial p'}{\partial x_k}}\right) \\
&+ \nu(\overline{u_k'\frac{\partial^2 u_i'}{\partial x_j \partial x_j}} + \overline{u_i'\frac{\partial^2 u_k'}{\partial x_j \partial x_j}})
\end{aligned}
\tag{20}
$$

$$
\nu\left(\overline{u_k'\frac{\partial^2 u_i'}{\partial x_j \partial x_j}} + \overline{u_i'\frac{\partial^2 u_k'}{\partial x_j \partial x_j}}\right) = \nu\frac{\partial^2 \overline{u_i' u_k'}}{\partial x_j \partial x_j} - 2\nu\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_k'}{\partial x_j}}
\tag{21}
$$

Turbulent kinetic energy (TKE)

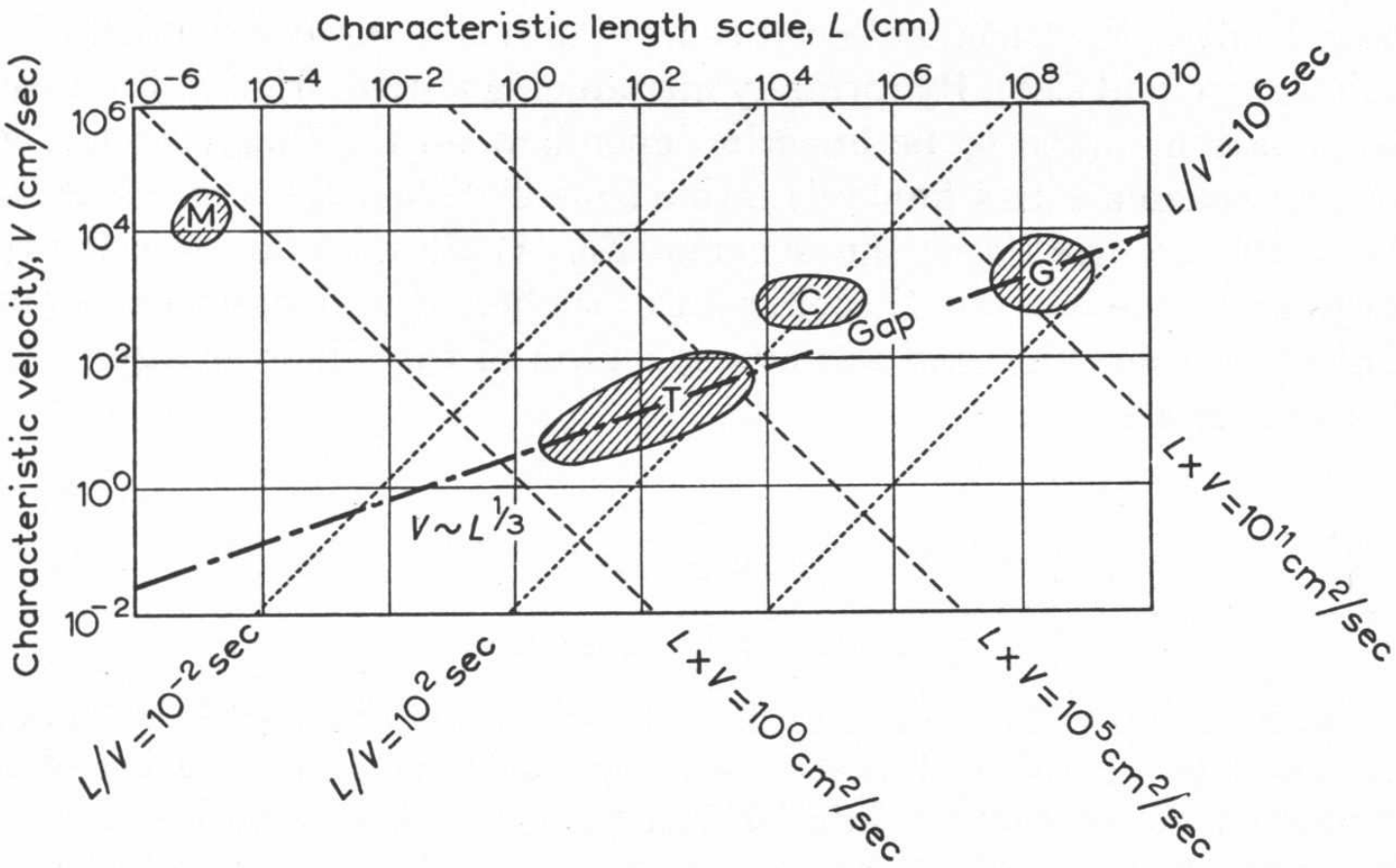
$\frac{1}{2}q^2 = \frac{1}{2}\overline{u_i' u_i'}$

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{1}{2}q^2\right) =$$

(22) $$-\overline{u_i' u_j'}\frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\frac{\overline{p' u_j'}}{\rho_{00}} + \frac{1}{2}\overline{u_i' u_i' u_j'}\right) - \frac{g}{\rho_{00}}\overline{\rho' u_3'} + \nu/2\frac{\partial^2 q^2}{\partial x_j \partial x_j} - \varepsilon$$

(23) $$\varepsilon = \nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$$

The Eulerian point of view



M = molecular unrest

T = ordinary turbulence

C = convective activity

G = general circulation

Kolmogorov (1941)

NSE without external forces, unbounded, for $\mathcal{R}_e \to \infty$, are invariant by scale change $x \to \lambda x$, $u_i \to \lambda^h u_i$

From TKE equation

(24) $$\frac{\mathrm{D}}{\mathrm{D}t}(\frac{1}{2}q^2) = -\overline{u_i' u_j'}\frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\frac{\overline{p' u_j'}}{\rho_{00}} + \frac{1}{2}\overline{u_i' u_i' u_j'}\right) - \lambda^{3h-1}\varepsilon$$

to get ε invariant it needs $h = 1/3$.

On dimensional basis

(25) $$[\varepsilon] = \frac{U^3}{L}\ ;\ U_l^2 \sim \varepsilon^{2/3} l^{2/3}$$

The limits of the inertial subrange $L_K \equiv \nu^{3/4}\varepsilon^{-1/4} < l < L$

Structure functions and spectra

The Eulerian picture:

(26) $$\overline{\Delta v^2} = \overline{[u(\mathbf{x}^{(2)}) - u(\mathbf{x}^{(1)})]^2} = 2[\overline{u^2} - R(l)] \text{ where } l = |\mathbf{x}^{(2)} - \mathbf{x}^{(1)}|$$

(27) $$\overline{\Delta v^2} = C_K \varepsilon^{2/3} l^{2/3} \text{ , } l < L$$

(28) $$\overline{\Delta v^2} = 2\overline{u^2} \text{ , } l \gg L$$

(29) $$[E(k)dk] = U_l^2 \text{ ; } E(k) \sim \varepsilon^{2/3} l^{2/3} k^{-1} \sim \varepsilon^{2/3} k^{-5/3}$$

The Lagrangian picture

(30) $$V_t^2 \equiv \overline{(V(t) - V(0))^2} \sim \varepsilon t$$

(31) $$[E(\omega)d\omega] = V_t^2 \text{ ; } E(\omega) \sim \varepsilon t \omega^{-1} \sim \varepsilon \omega^{-2}$$

Horizontal homogeneity

(32) $$\frac{\partial \overline{u}_i}{\partial t} = \varepsilon_{ij3} f(\overline{u}_j - u_{gj}) + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial \overline{u}_i}{\partial x_3} - \overline{u_i' u_3'}\right), \; i = 1, 2$$

(33) $$\frac{\partial \overline{\vartheta}}{\partial t} = \frac{\partial}{\partial x_3}\left(\chi \frac{\partial \overline{\vartheta}}{\partial x_3} - \overline{u_3' \vartheta'}\right)$$

If the fluxes are dependent on height $z \equiv x_3$, the boundary layer must be unsteady and the wind rotates.

The constant flux layer (surface layer)

(34) $$\nu \frac{\mathrm{d}^2 \overline{u}}{\mathrm{d}z^2} + \frac{\mathrm{d}(-\overline{u'w'})}{\mathrm{d}z} = 0$$

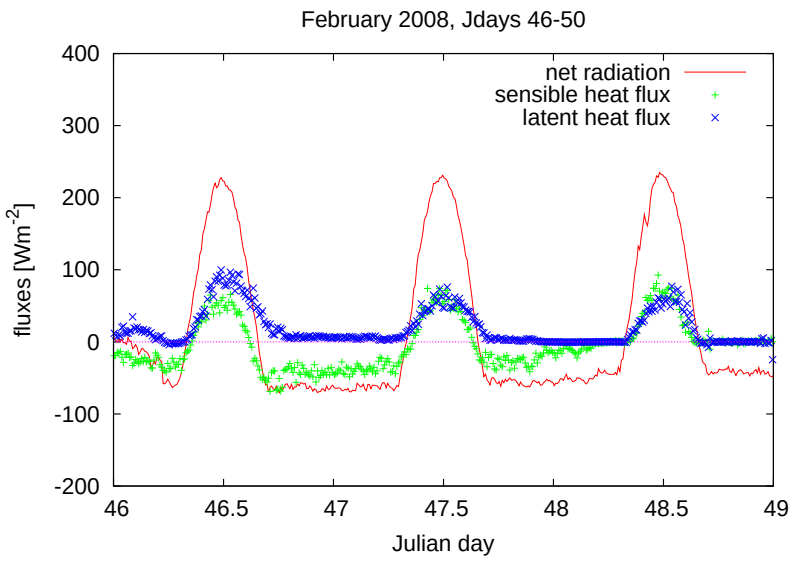
(35) $$\chi \frac{\mathrm{d}^2 \overline{\vartheta}}{\mathrm{d}z^2} + \frac{\mathrm{d}(-\overline{\vartheta' w'})}{\mathrm{d}z} = 0$$

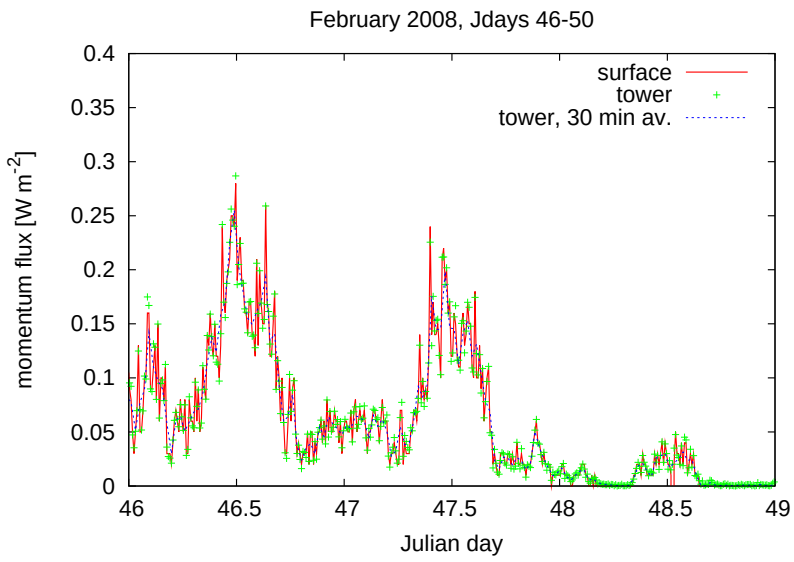
(36) $$-\overline{u'w'} \frac{\mathrm{d}\overline{u}}{\mathrm{d}z} + \frac{g}{\vartheta_{00}} \overline{\vartheta' w'} - \frac{\mathrm{d}}{\mathrm{d}z} \left(\frac{\overline{p'w'}}{\rho_{00}} + \frac{1}{2} \overline{u_i' u_i' w'} \right) + \frac{\nu}{2} \frac{\mathrm{d}^2\, q^2}{\mathrm{d}z^2} - \varepsilon = 0$$

(37) $$\mathcal{R}_f = \frac{\frac{g}{\vartheta_{00}} \overline{w'\vartheta'}}{\overline{u'w'} \frac{\mathrm{d}\overline{u}}{\mathrm{d}z}} \; ; L = -\frac{u_*^3}{\kappa g \overline{w'\vartheta'}|_0 / \vartheta_{00}}$$

(38) $$-\overline{u'w'} \frac{\mathrm{d}\overline{u}}{\mathrm{d}z} (1 - \mathcal{R}_f) - \frac{\mathrm{d}}{\mathrm{d}z} \left(\frac{\overline{p'w'}}{\rho_{00}} + \frac{1}{2} \overline{u_i' u_i' w'} \right) - \varepsilon \simeq 0$$

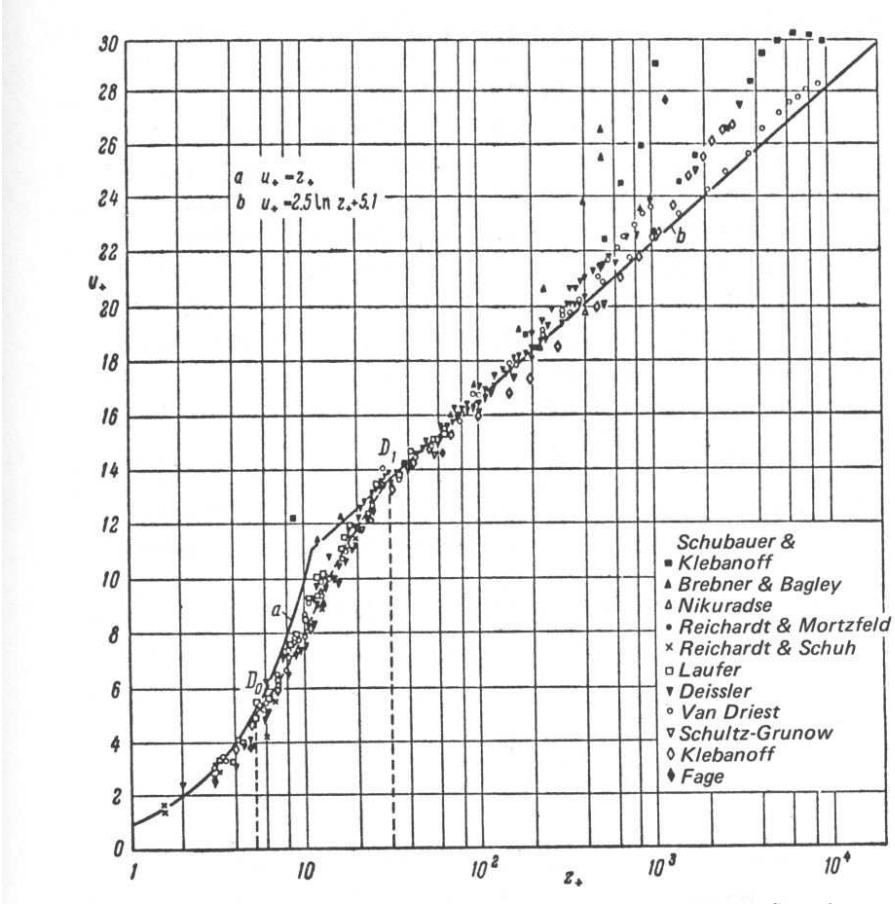
Momentum and heat fluxes





The surface layer dynamics is driven by the balance between momentum and sensible heat fluxes: Monin-Obukhov similarity theory (MOST)

NBL: $1/L = 0$


a $u_+ = z_+$
b $u_+ = 2.5 \ln z_+ + 5.1$
u_+
z_+
D_0
D_1
a
b
Schubauer & Klebanoff
Brebner & Bagley
Nikuradse
Reichardt & Mortzfeld
Reichardt & Schuh
Laufer
Deissler
Van Driest
Schultz-Grunow
Klebanoff
Fage

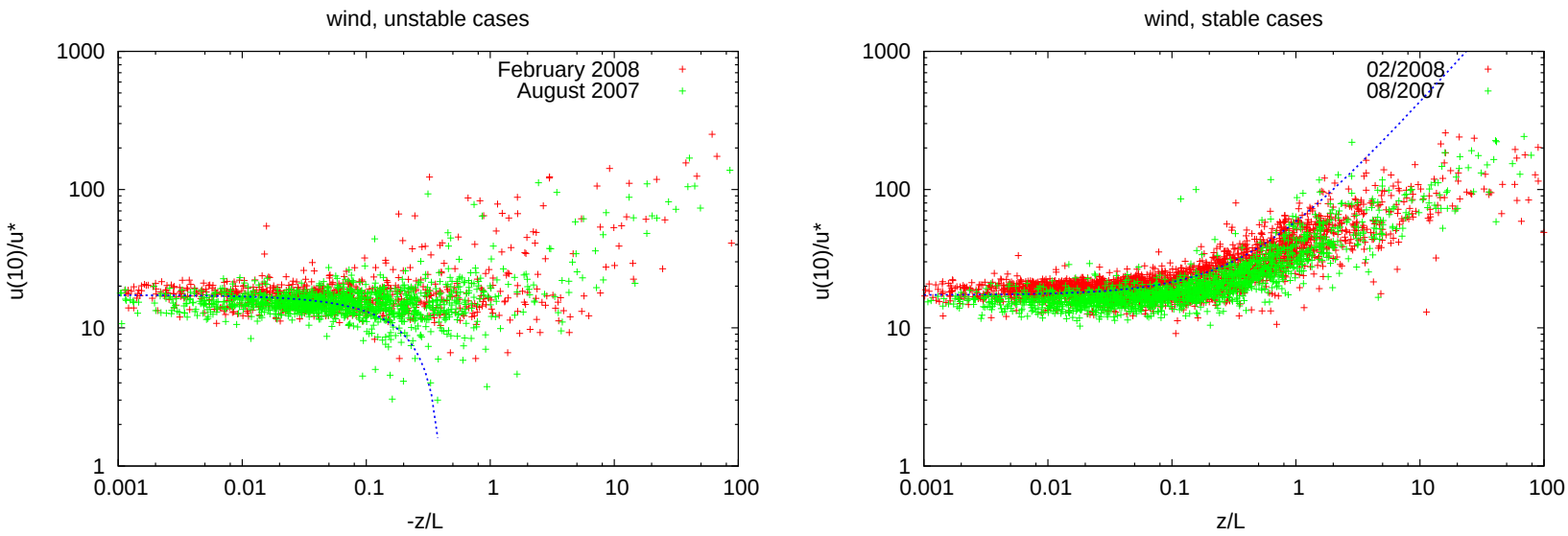
NBL: $1/L = 0$

(39) $$-\overline{u'w'} = u_*^2$$

(40) $$\frac{\mathrm{d}\overline{u}}{\mathrm{d}z} \propto \frac{u_*}{z} \; ; \overline{u}(z) = \frac{u_*}{\kappa} \ln \frac{z}{z_0}$$

where $\kappa = 0.4$

Changing stability



The log-linear relationship fails to fit the cases with large stability.

QNBL: $|1/L| << 1$

(41) $$\frac{\mathrm{d}\overline{u}}{\mathrm{d}z} = \frac{u_*}{\kappa z}\Phi_m(\zeta)$$

where $\zeta = z/L$.

(42) $$\Phi_m(\zeta) = 1 + \alpha_m\zeta + \mathcal{O}(\zeta)^2$$

to the first order

(43) $$\overline{u}(z) \cong \frac{u_*}{\kappa}\left(\ln\frac{z}{z_0} + \alpha_m\frac{z - z_0}{L}\right)$$

The temperature scale

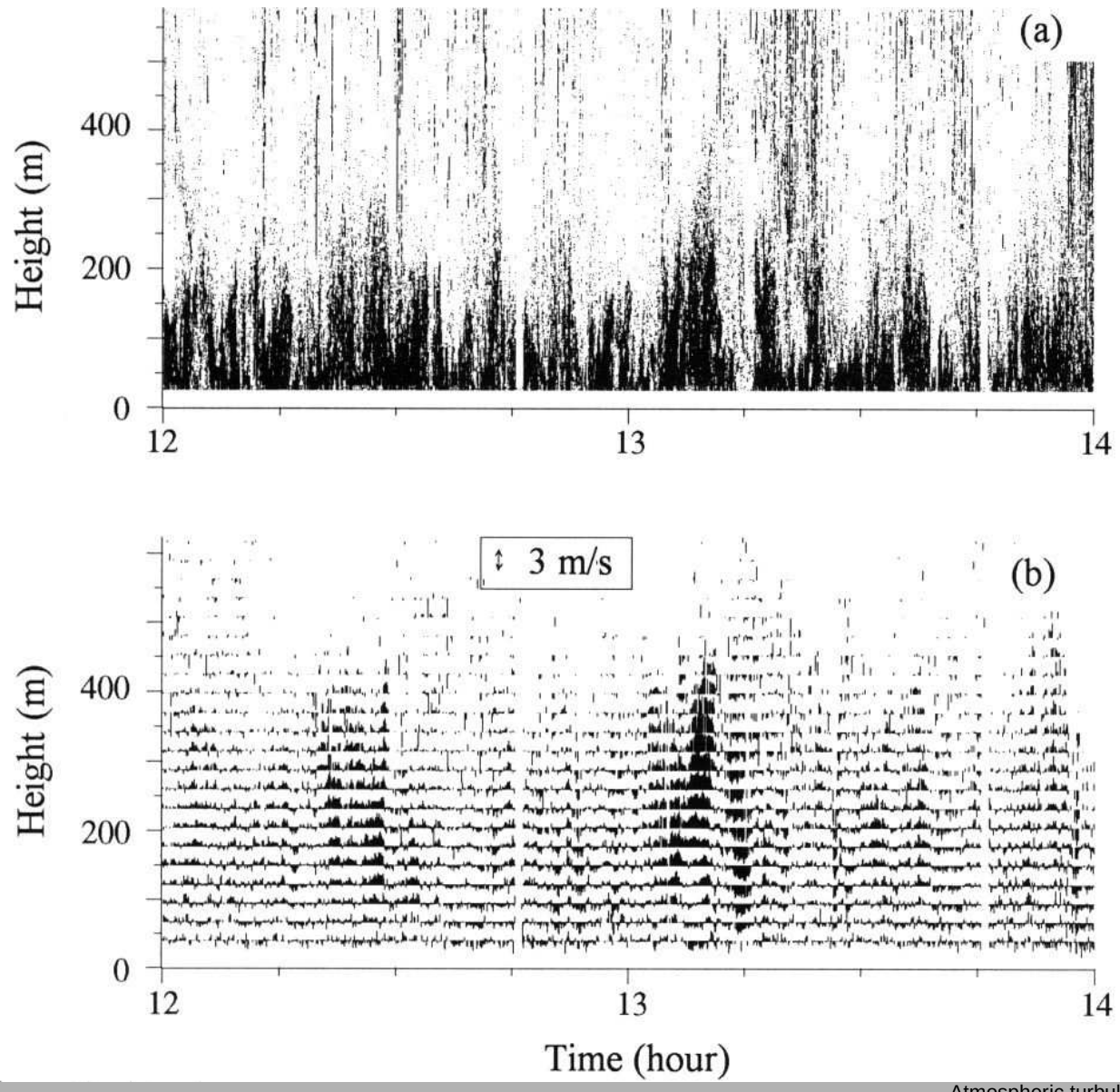
(44) $$\vartheta_* = \frac{\overline{w'\vartheta'}|_0}{u_*}$$

(45) $$\frac{\mathrm{d}\overline{\vartheta}}{\mathrm{d}z} = -\frac{\vartheta_*}{\kappa z}\Phi_h(\zeta)$$

(46) $$\Phi_h = \alpha_{h0} + \alpha_{h1}\zeta$$

where α_m, α_{h0} and α_{h1} are to be determined from observations.

CBL: $1/L < 0$


(a)
Height (m)
400
200
0
12
13
14
3 m/s
(b)
Height (m)
400
200
0
12
13
14
Time (hour)

Free convection scales: $1/L \ll -1$

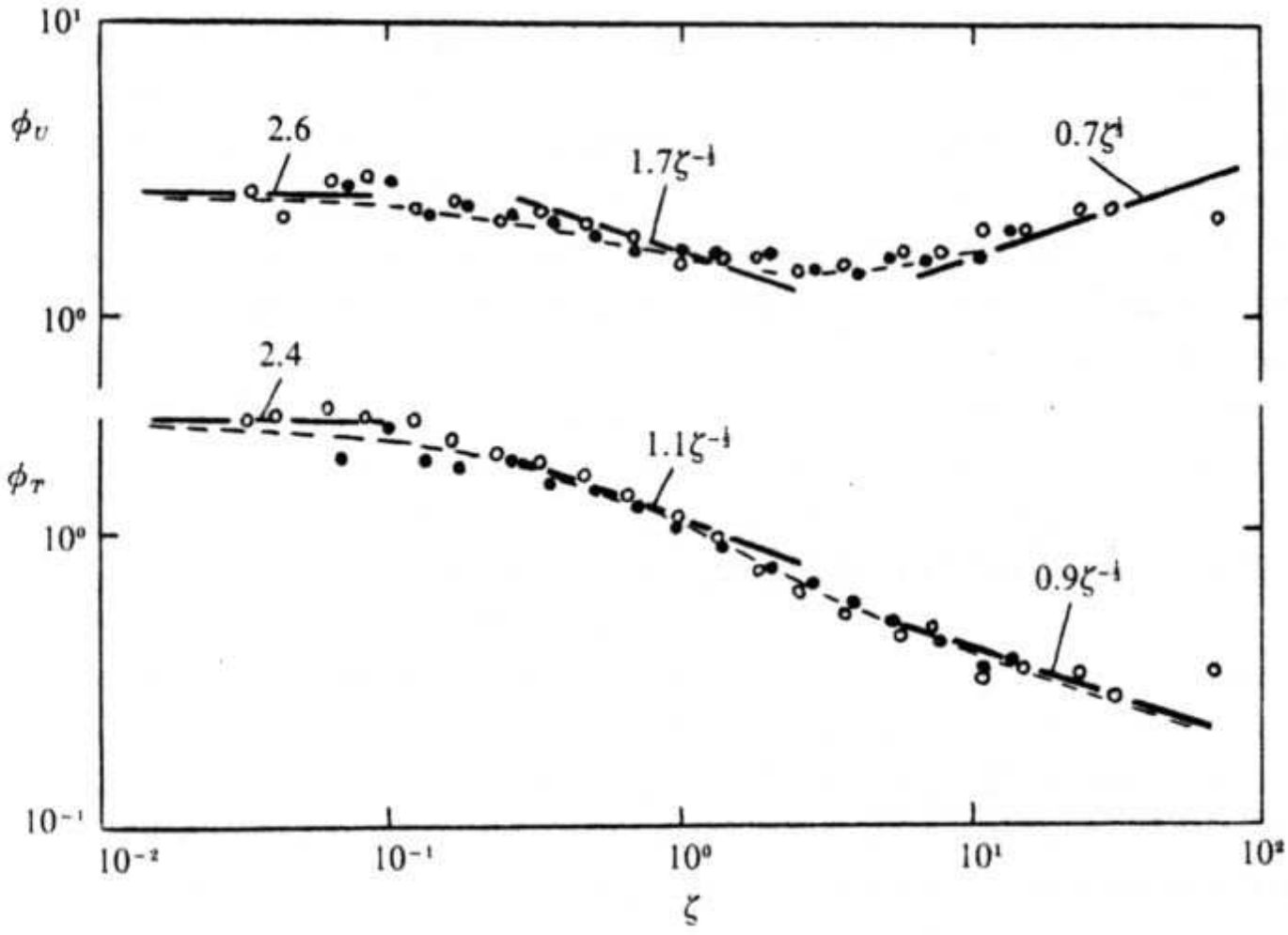
(47) $$w_* = \left(\frac{g}{\vartheta_{00}} \overline{w'\vartheta'}|_0 z \right)^{1/3}$$

(48) $$\vartheta_{**} = \frac{\overline{w'\vartheta'}|_0}{w_*}$$

(49) $$\frac{\mathrm{d}\overline{u}}{\mathrm{d}z} \approx \frac{w_*}{z} = (g/\vartheta_{00}\overline{w'\vartheta'}|_0)^{1/3} z^{-2/3}$$

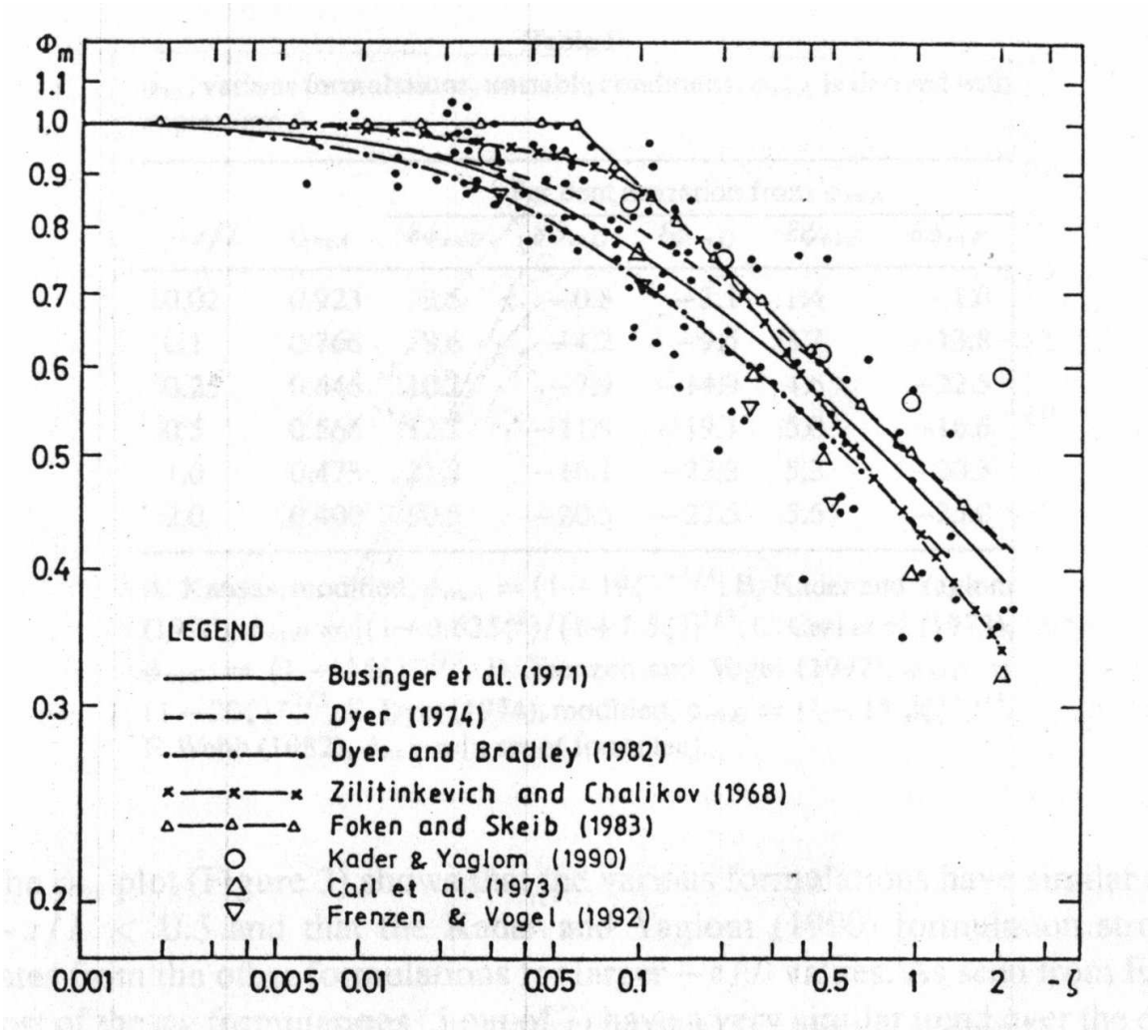
(50) $$\frac{\mathrm{d}\overline{\vartheta}}{\mathrm{d}z} \approx -\frac{\vartheta_{**}}{z} = -(g/\vartheta_{00})^{-1/3} (\overline{w'\vartheta'}|_0)^{2/3} z^{-4/3}$$

CBL: $\overline{u}$ and $\overline{\vartheta}$ gradients

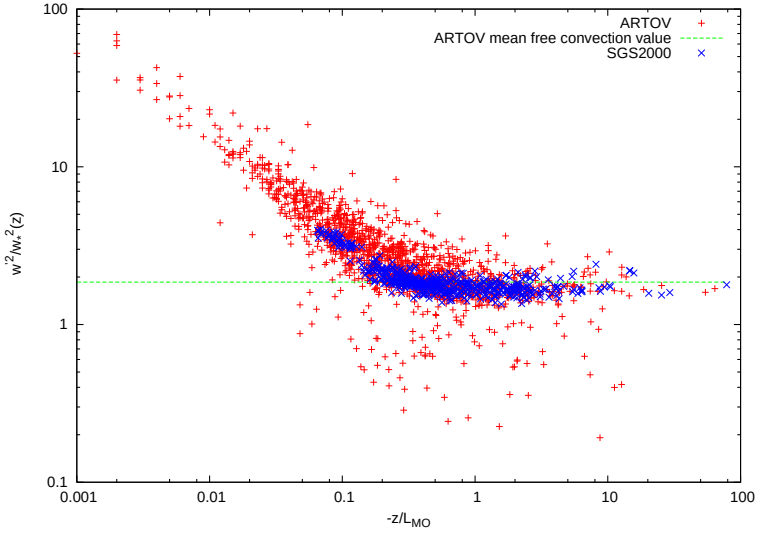


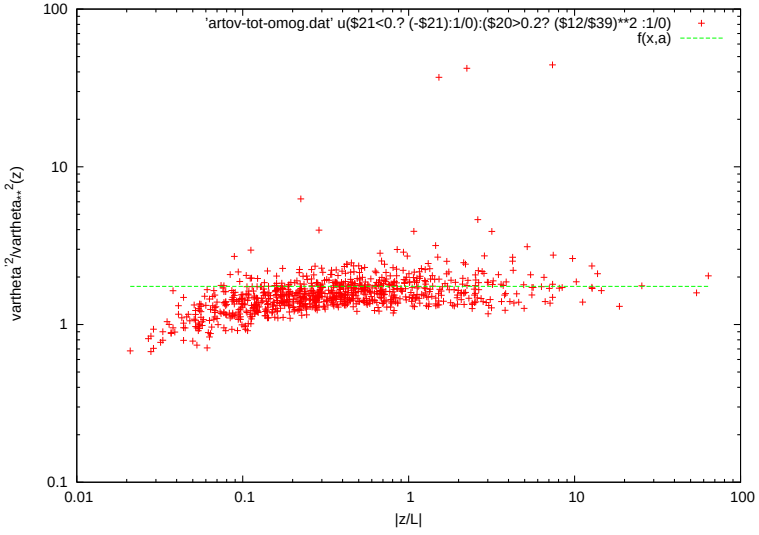
Remember the different definitions of scales.

CBL: $\overline{u}$ and $\overline{\vartheta}$ gradients

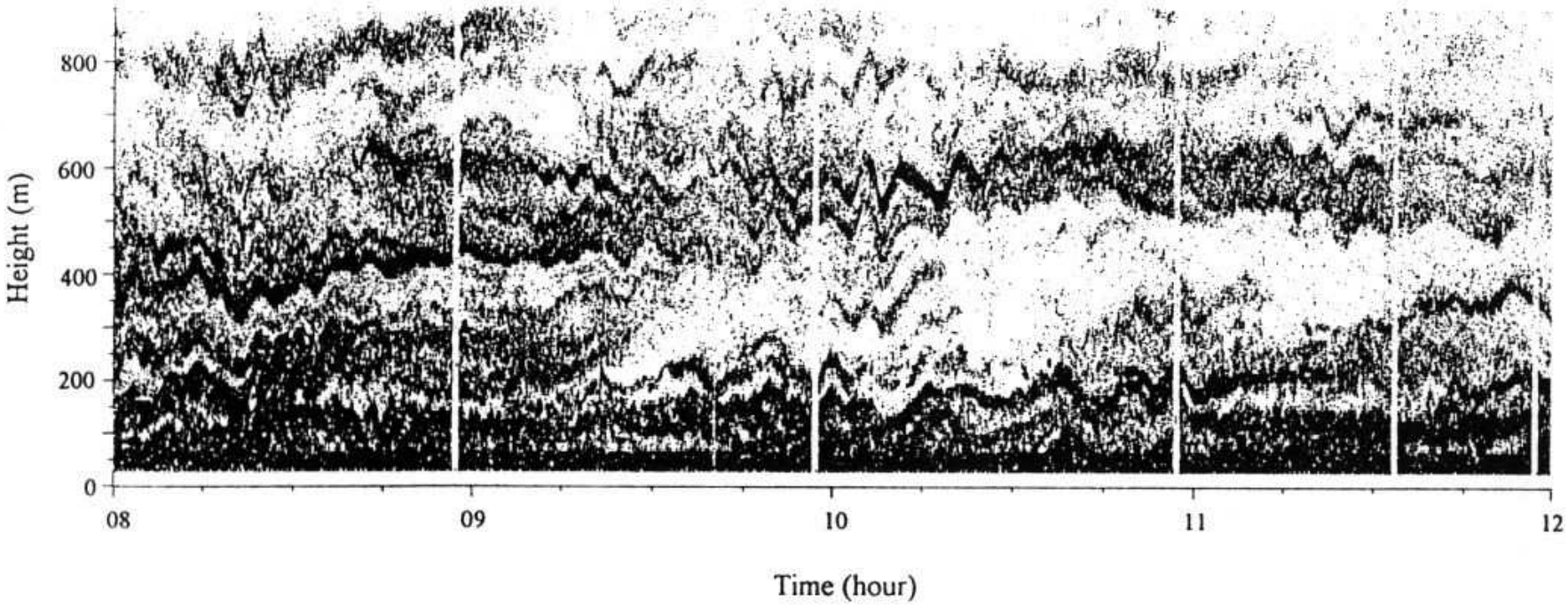


Free convection scaling for variances





SBL: $1/L > 0$



The critical Richardson number

In the surface layer:

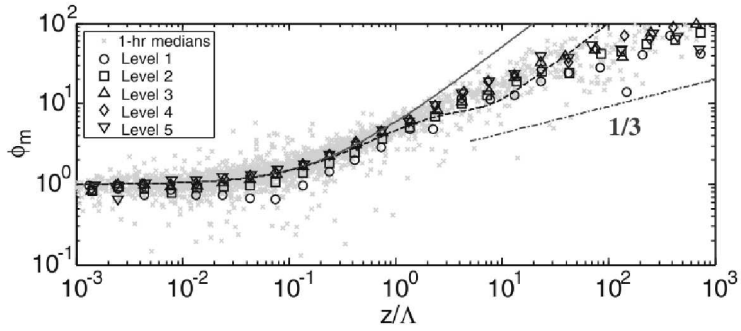
(51) $$\mathcal{R}_{fc} \equiv -\frac{g}{\vartheta_{00}} \frac{\overline{w'\vartheta'}}{u_*^2 \, \mathrm{d}\overline{u}/\,\mathrm{d}z} = \frac{\zeta}{\Phi_m(\zeta)}$$

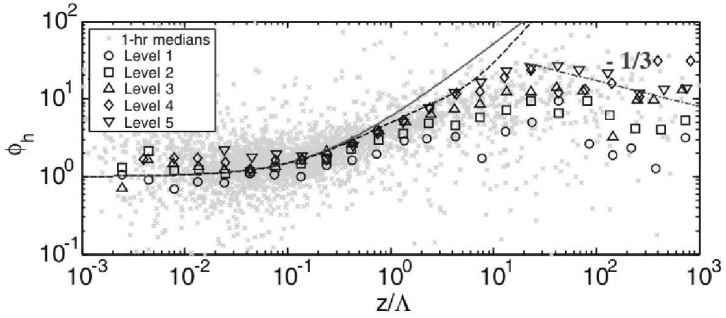
and the asymptotic expression for the mean velocity gradient

(52) $$\frac{\mathrm{d}\overline{u}}{\mathrm{d}z} = \frac{u_*}{\kappa L \mathcal{R}_{fc}}$$

Is $\mathcal{R}_{fc}$ a true limit which cannot be exceeded?

Local scaling for mean variables

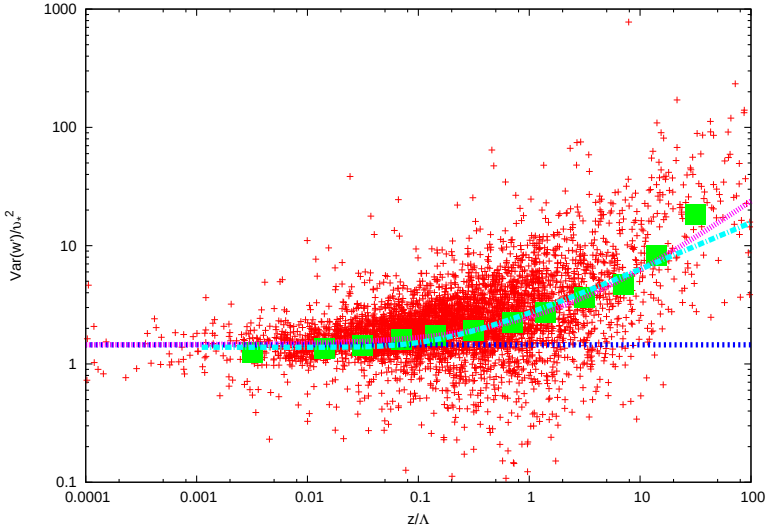


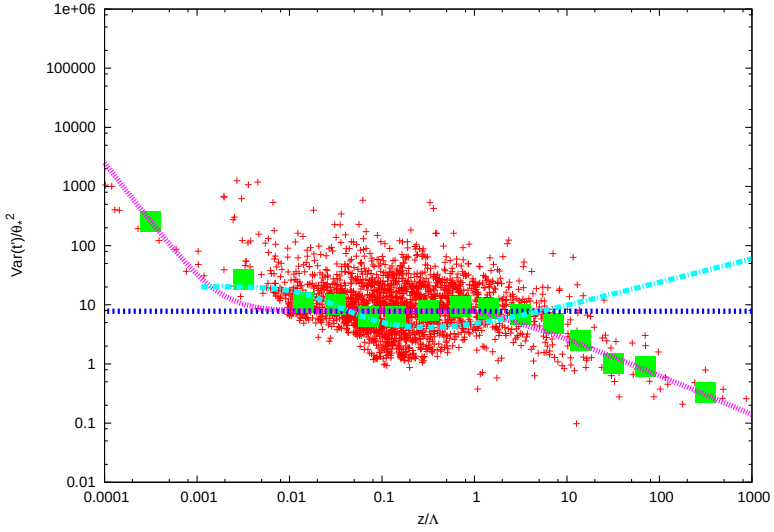


Now Λ is the Obukhov lenght computed using the local values of the fluxes.

For $z/\Lambda > 1$ the data depart from the log-linear expression.

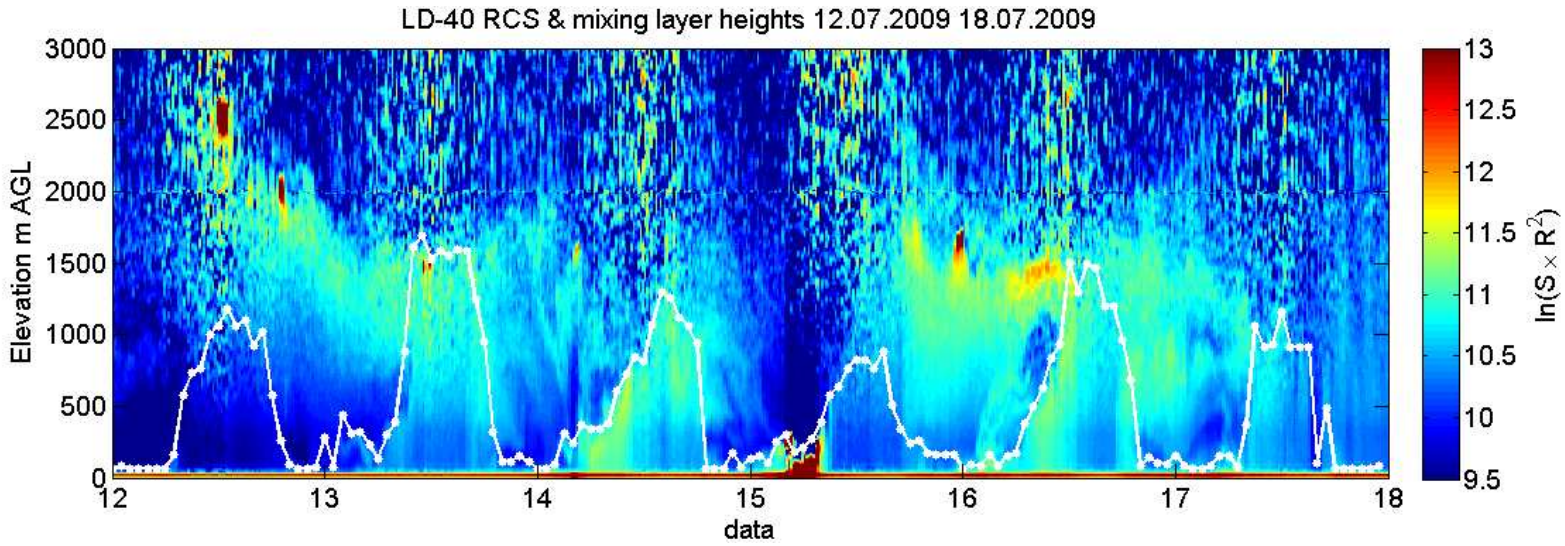
Local scaling for variances





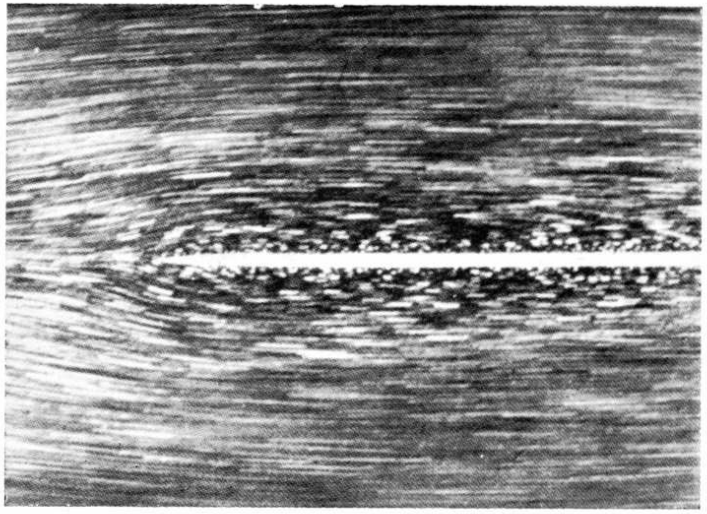
PBL evolution at SPC

What occurs above the constant flux layer?



courtesy Gian Paolo Gobbi

Prandtl, 1904: NBL on a flat plate



$lU/\nu = 3$

from Batchelor (1967). The height of non-rotating, viscous NBL:

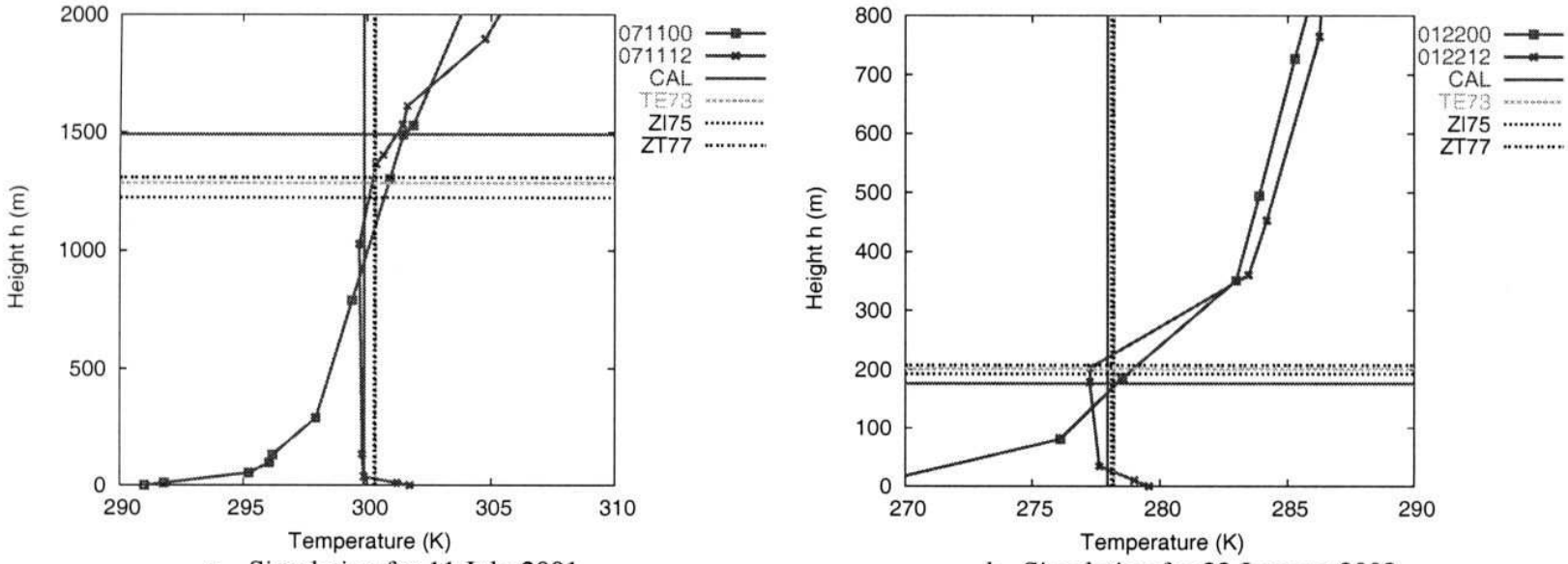
(53) $$h = \int_0^\infty \left(1 - \frac{\overline{u}}{U}\right) \mathrm{d}z \; ; h \propto \sqrt{\frac{x}{U\nu}}$$

where $U = \lim_{z\to\infty} \overline{u}(z)$

In a rotating reference system:

(54) $$h = \alpha \frac{u_*}{f}$$

CBL height



a. Simulation for 11 July 2001

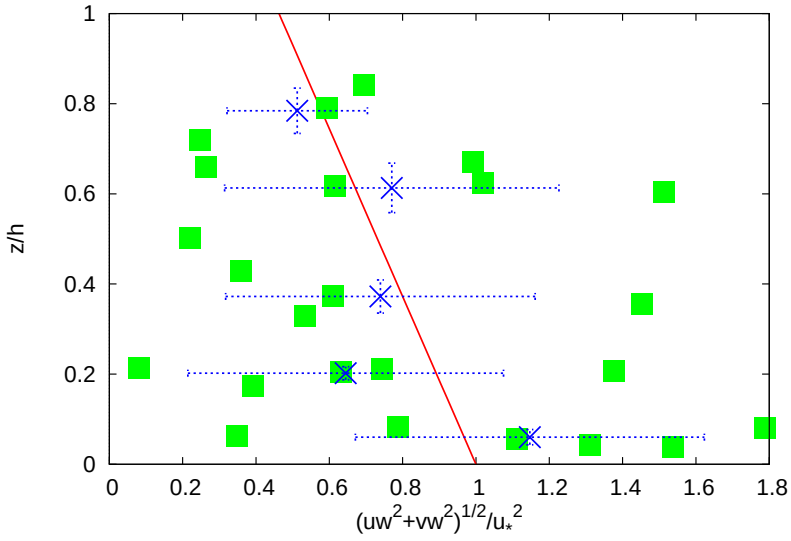
b. Simulation for 22 January 2002

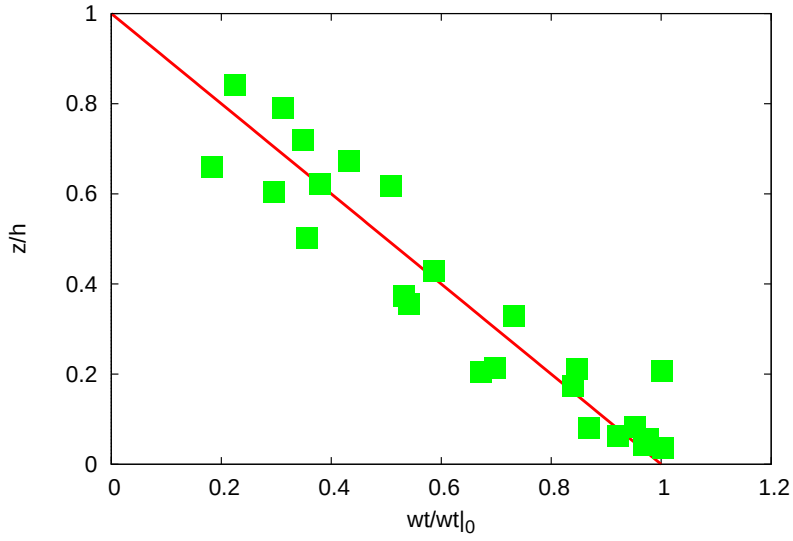
from Villani et al. (2005). The encroachment model reads:

(55)
$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\overline{w'\vartheta'}|_0}{\Gamma h}$$

where Γ is the nocturnal potential temperature gradient.

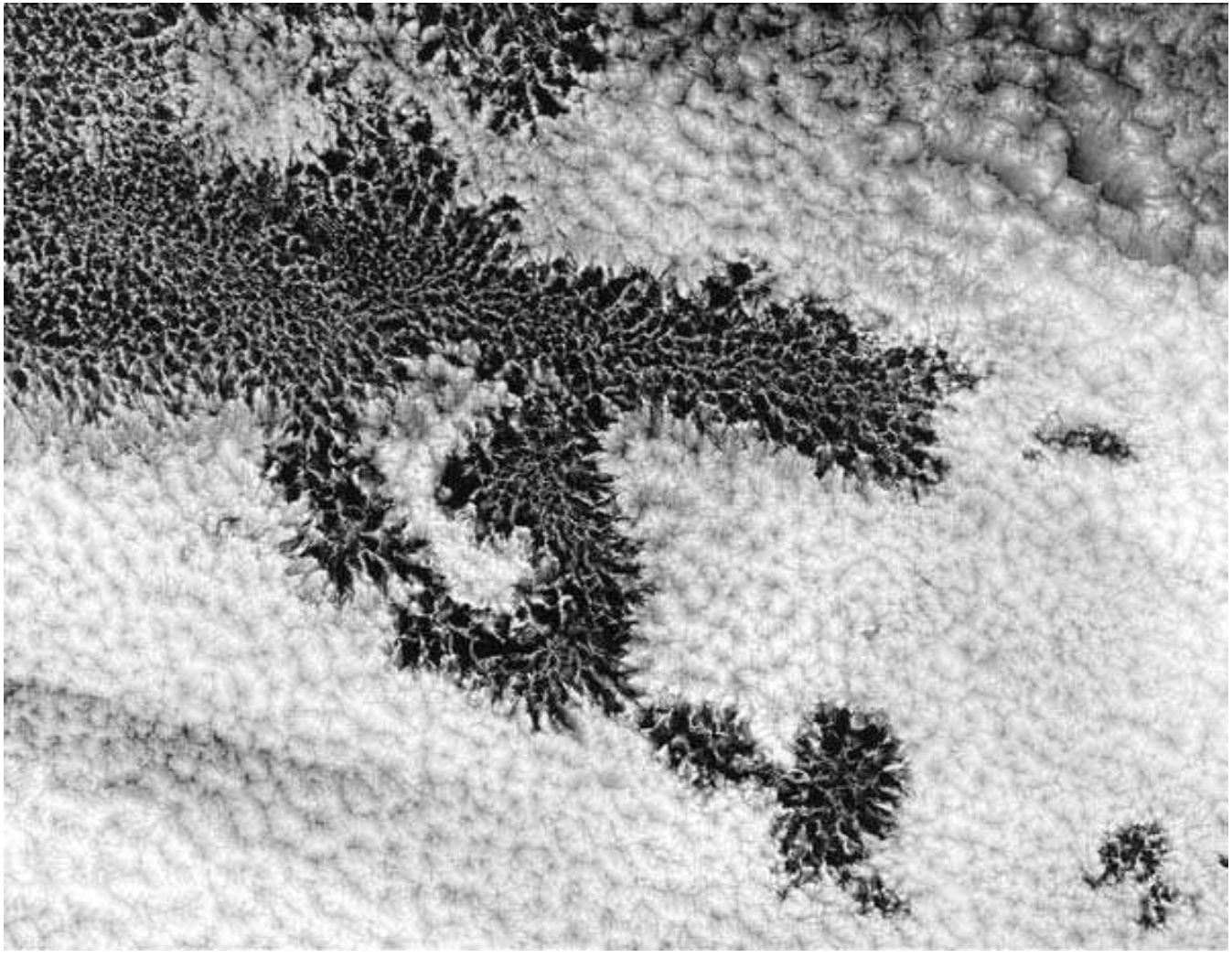
cloud-free CBL





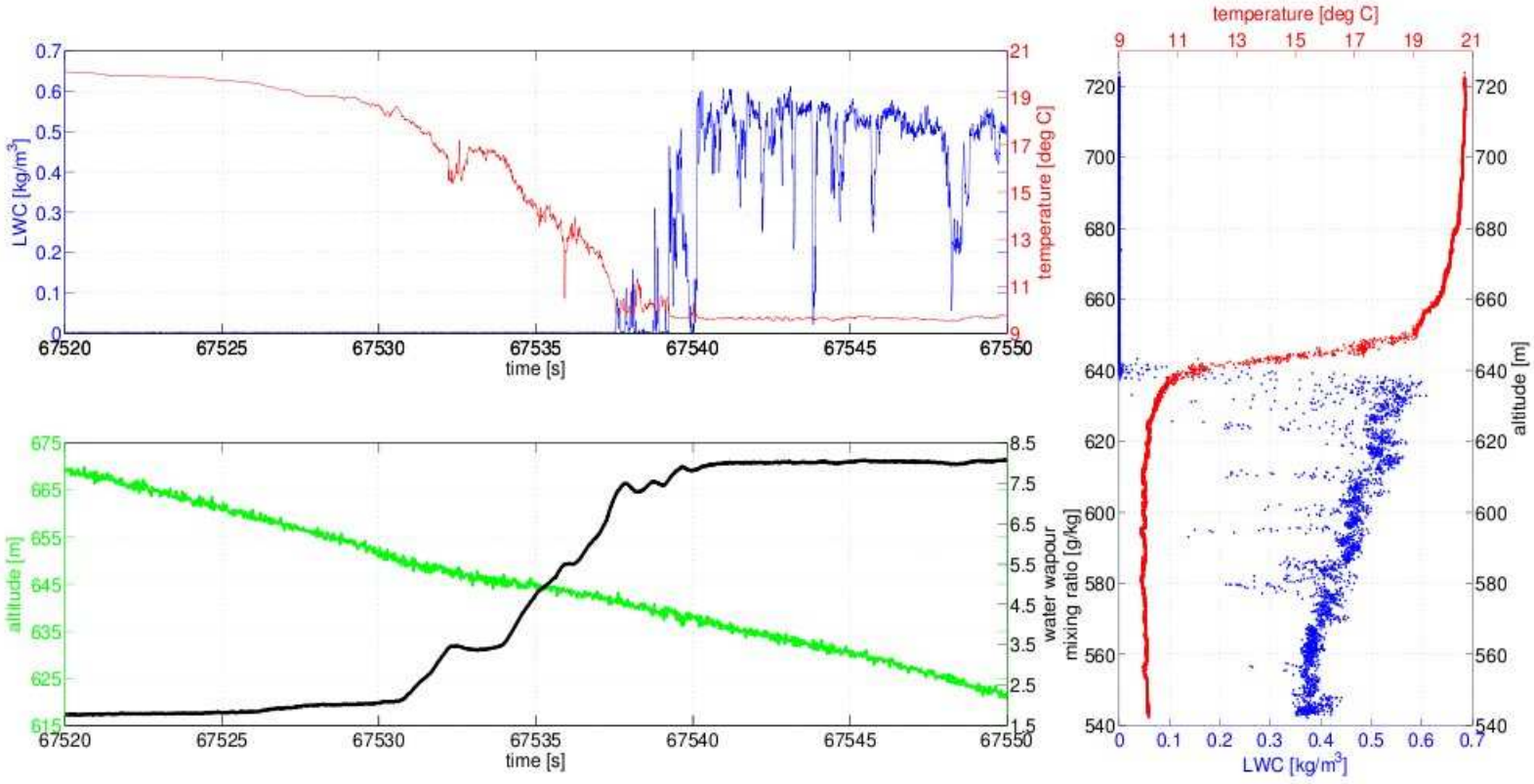
data by Hartmann, project ARTIST. The CBL height is well defined in terms of heat flux, but not in terms of momentum flux.

clouds at the CBL top



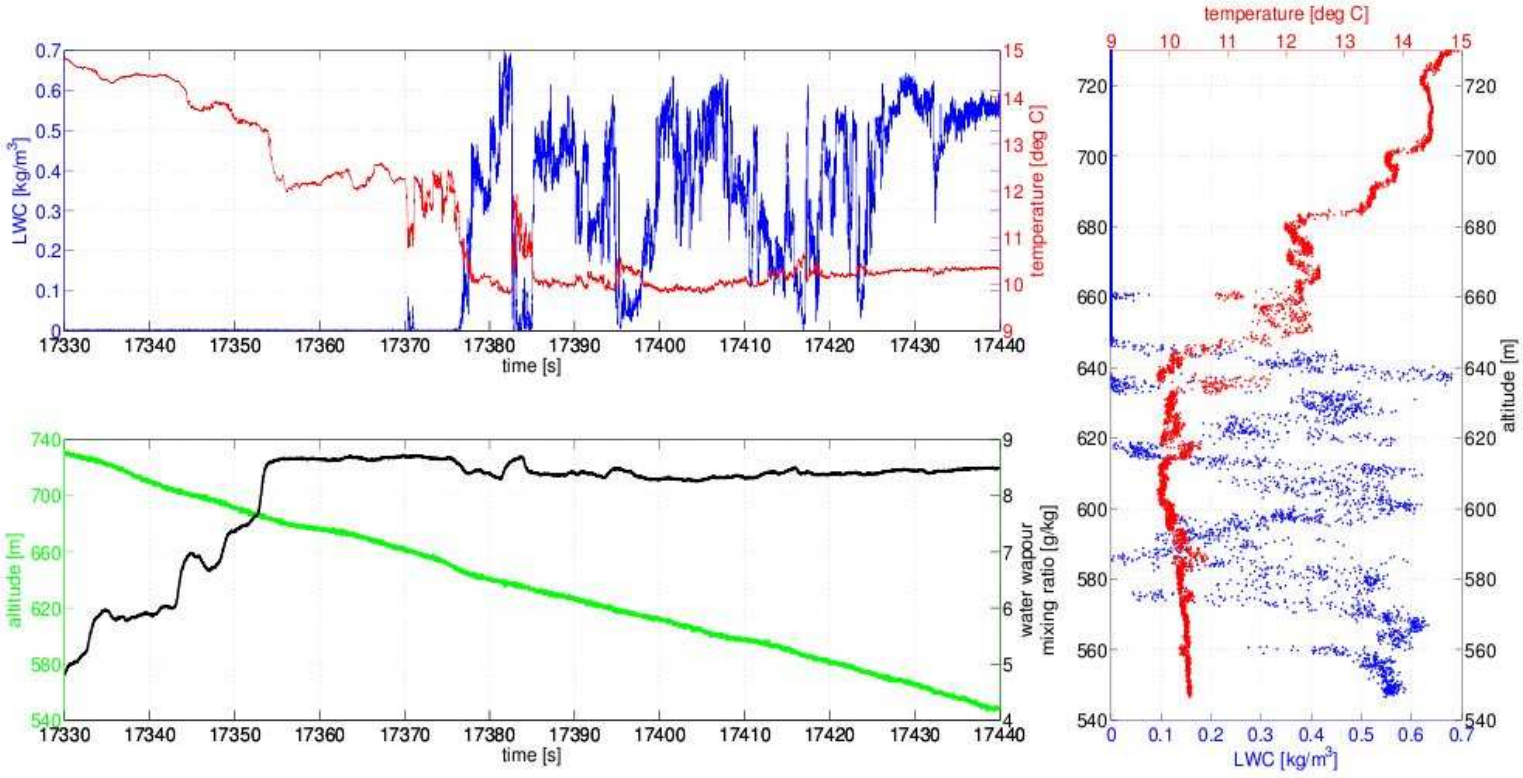
from Stevens (2005). Open cells have the internal part dark, closed cells are almost white.

A case of mixing and downdraft: TO10



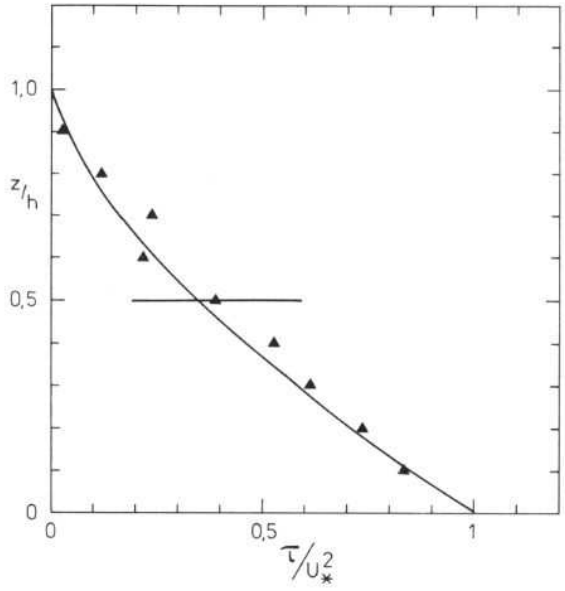
From Malinowsky et al.(2011)

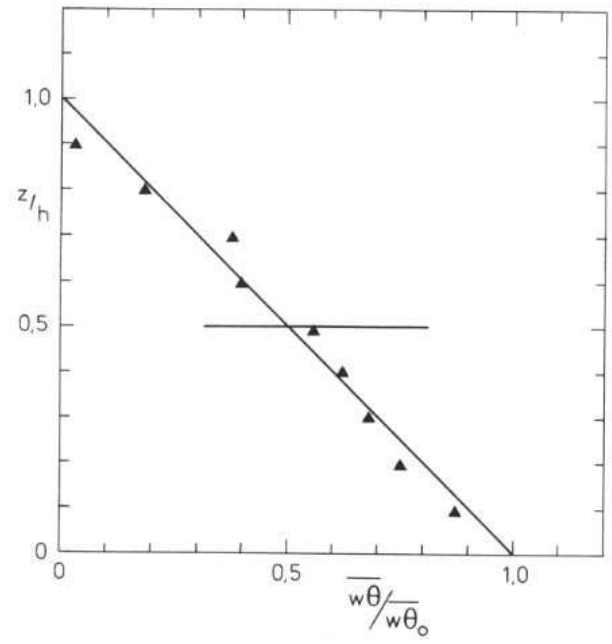
Different mixing: TO13



From Malinowsky et al.(2011)

Nieuwstadt, 1984: $\mathcal{R}_{fc} \simeq 0.2$



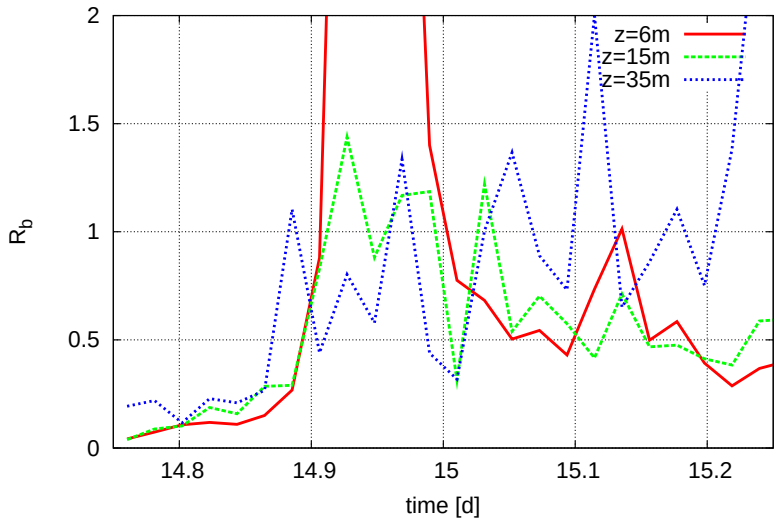


A simple formula:

(56)
$$h = \gamma_c \left(\frac{u_* L}{f} \right)^{1/2}$$

with $\gamma_c = (\sqrt{3}\kappa\mathcal{R}_{fc})^{1/2} \approx 0.4$

More stable SBL: bulk Richardson number

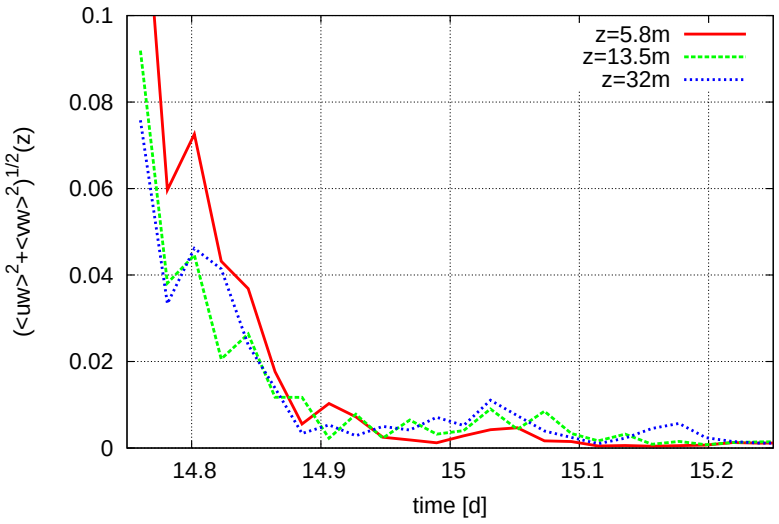


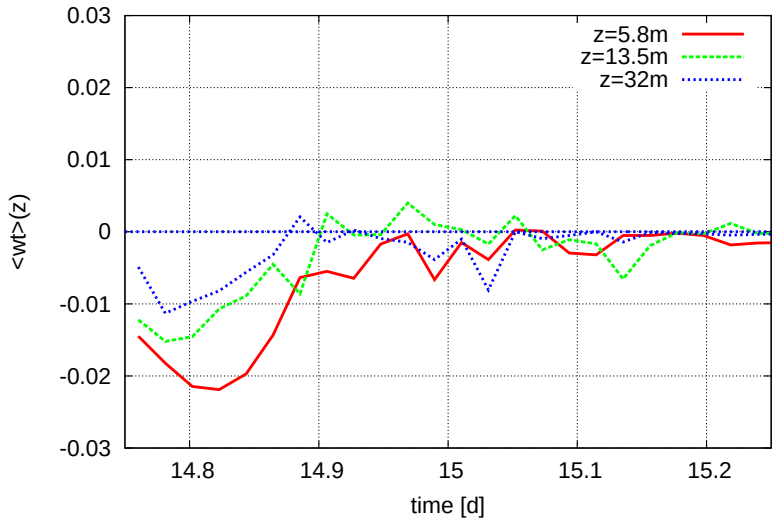
Bulk Richardson number

(57)
$$\mathcal{R}_b = \frac{g}{\vartheta_{00}} \frac{\Delta\overline{\vartheta}}{z_2 - z_1} \frac{(z_3 - z_0)^2}{\overline{u}^2}$$

as function of time, during the night of 15 sept. 1999, CIBA site, Spain. Red line: $z \simeq 6m$, green line $z \simeq 14m$, blue line $z \simeq 32m$.

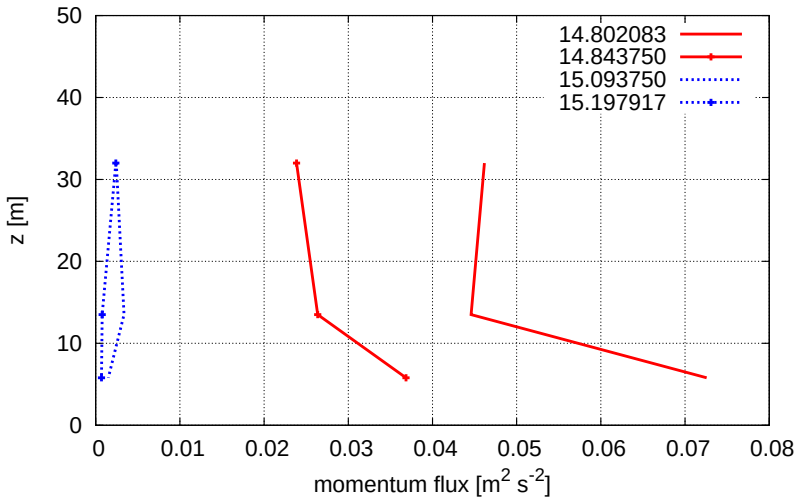
More stable SBL: turbulent fluxes

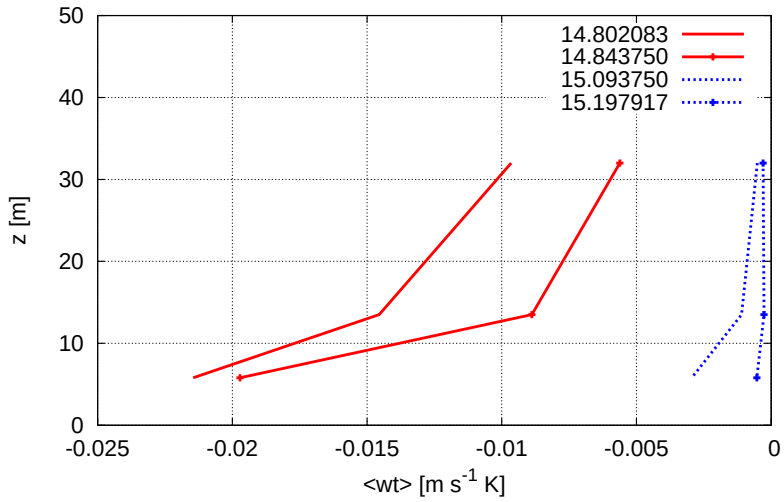




As the fluxes become negligible, the stable boundary layer loses its characterisation (MOST does not hold).

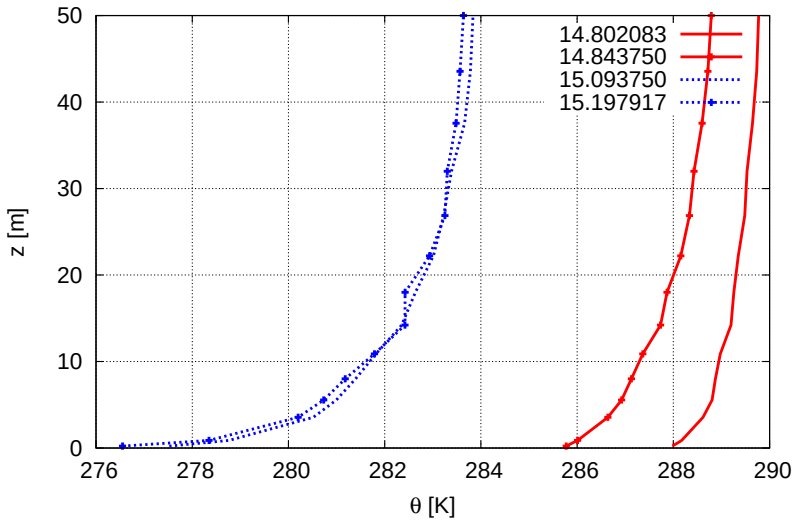
Vertical structure of the fluxes

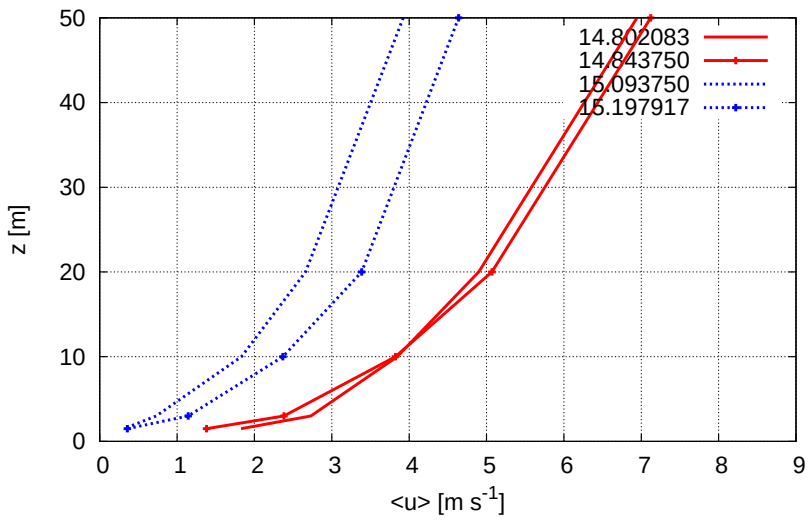




Quite large vertical gradients in condititions of moderate stability (fluxes decrease with height); almost constant negligible fluxes otherwhise.

Mean profiles





Note the large temperature gradient near the ground for the cases with negligible fluxes.